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Contents

1 Plenary talks

Graph coloring: Connecting semi-online algorithms to new algorithmic paradigms.

Magnús Halldórsson
Reykjavik University, Iceland

The aim of this talk is to draw connections between online algorithms and other more recent models of computation. This includes streaming algorithms, distributed models, and sublinear-time algorithms.

We revisit the vertex coloring problem: assigning the nodes of graph colors so that adjacent nodes receive different colors, where the objective is to use few colors. In the online context, the algorithm receives in each round a new node with edges to previous vertices and must then irrevocably assign the node a color consistent with the previous assignments.

It is known since 1990 that the online coloring problem is extremely hard: there are graphs on n vertices that require *log n* colors for which any online algorithm uses at least *n/log n* colors. This holds under numerous relaxations: lookahead, buffering, (limited amount of) recoloring, and even if the adversary reveals its coloring after each step. Furthermore, it holds even if the whole graph is known in advance, but not the mapping of presented vertices to the original ones. We conclude that what makes the problem so hard is the irrevocability requirement: that the color assigned cannot be modified later. It is natural to ask what performance guarantees can be achieved if one allows each node to be recolored a fixed number of times. There are recent results that come quite close to characterizing this bicriteria approximation.

In the streaming setting, the algorithm only needs to determine the coloring at the end, but is constrained by the amount of space allowed (and in particular can only store a small fraction of the whole graph). In semi-streaming, we allow the algorithm to store polylogarithmic number of bits per vertex of the graph. Recent results show that Delta+1-coloring can be solved in semi-streaming model, i.e., using Delta+1 colors where Delta is the maximum degree. Note that this is trivial online but quite non-trivial in all the constrained models mentioned above. Interestingly, Delta-coloring has also been shown to be solvable in semi-streaming, while no online algorithm can guarantee to solve that problem.

Finally, we mention a specific model that combines both online and streaming aspects. We start with an empty graph with a trivial solution. The edges then arrive in a stream and we must at all times maintain a valid solution. We have therefore an online setting where the solution is decreasing by time, rather than the ordinary case of being constructed in a piecemeal fashion. We may also require that the space use be limited, thus capturing streaming constraints. Interestingly, this model allows for non-trivial approximations of the maximum independent set problem, which is impossibly hard in the traditional online setting.

In conclusion, new models will continue to be introduced to capture technological changes. These models generally have in common that the algorithm is restricted somehow, with "one hand tied behind the back". It is worthwhile to explore the commonalities of these models, as well as what separates them. This is certain to give classical aspects like the online dimension a still longer lease on life.

On couple of task scheduling

GÁBOR GALAMBOS
University of Szeged, Department of Applied Informatics

The coupled task problem can be defined as follows: We are given n jobs. Each of them consisting of two distinct tasks. The sequence of tasks are fixed and also a fixed length of delay passes between the consecutive tasks. The i^{th} job is denoted by a triple (a_i, L_i, b_i) where a_i, L_i, b_i denote the processing time of the first task, the (exact) delay time between the tasks, and the processing time of the second task, resp. The second task of the job i must be started with an exact delay L_i after the completion of the first task. The aim is to schedule the n jobs on one machine in such a way that no jobs are overlapped, no preemption is allowed, and some objective function has to be minimized. We will consider those problems where the objective function is either to minimize the makespan, or to minimize the total completion time.

The aim of this talk to show different techniques have been used while the researchers investigated these problems. We will present techniques how can we prove that a given CTP belongs to the NP-class of problems, we also consider some – not always trivial – polynomial time algorithms which confirm that the CTP problem with given parameters belongs to the problem-class P. Finally, we show some typical techniques that we can use while analyse approximation algorithms from worst case point of view.

Models of Scheduling on Parallel Machines under Resource Constraints

VITALY STRUSEVICH
UK

Scheduling problems of processing jobs on parallel identical machines under a constraint that some jobs at any time of their execution need one unit of one of the additional non-renewable resources are considered.

Approximation algorithms and schemes for the problem of minimizing the makespan with no preemption allowed are discussed.

If preemption is allowed, a range of the necessary and sufficient conditions is reported for the existence of a feasible schedule in which jobs have release dates and all jobs must be completed before a given common deadline.

Monotone Dynamic Programming: From Theory to Approximation Schemes

Nir Halman
Bar-Ilan University, Israel

We propose to view a dynamic program (DP) as a collection of functions (whose argument is the state space), as opposed to a collection of tables.

We consider the special case where the functions the DP consists of are monotone. We design a framework to derive fully polynomial time approximation schemes (FPTASes) to monotone DPs that satisfy a certain set of conditions.

To get such a framework, we introduce 3 main building blocks: K-approximation functions, K-approximation sets and the calculus of K-approximation functions.

Our framework is quite general, and as such has been applied to derive first/faster FPTASes to a wealth of problems in various fields of research such as theoretical computer science, operations research, operations management, scheduling theory, to name a few.

One can think of such a framework as a tool that delegates algorithm design and running time analyses to DP (re)formulations.

Online Packet Scheduling

Jiří Sgall
Charles University, Prague, Czech Republic

We survey some older and new results on online packet scheduling with deadlines.

In this problem, the goal is to schedule transmissions of packets that arrive over time in a network switch and need to be sent across a link. Each packet has a deadline, representing its urgency, and a non-negative weight, that represents its priority. Only one packet can be transmitted in any time slot, so if the system is overloaded, some packets will inevitably miss their deadlines and be dropped. In this scenario, the natural objective is to compute a transmission schedule that maximizes the total weight of packets that are successfully transmitted. The problem is inherently online, with the scheduling decisions made without the knowledge of future packet arrivals.

We focus on the optimal deterministic φ-competitive algorithm. On the way we also present the results on greedy algorithms and randomized algorithms and present some open problems.

2 Contributed talks

2.1 Monday:

On the number of maximal independent sets: From Moon-Moser to Hujter-Tuza

Balázs Patkós
Alfréd Rényi Institue of Mathematics, Hungary

co-author: Cory Palmer, University of Montana, United States

We connect two classical results in extremal graph theory concerning the number of maximal independent sets. The maximum number $\mathrm{mis}(n)$ of maximal independent sets in an n-vertex graph was determined by Miller and Muller and independently by Moon and Moser. The maximum number $\mathrm{mis}_{\triangle}(n)$ of maximal independent sets in an n-vertex triangle-free graph was determined by Hujter and Tuza. We give a common generalization of these results by determining the maximum number $\mathrm{mis}_t(n)$ of maximal independent sets in an n-vertex graph containing no induced triangle matching of size $t + 1$. This also improves a stability result of Kahn and Park on $\mathrm{mis}_{\triangle}(n)$. Our second result is a new (short) proof of a second stability result of Kahn and Park on the maximum number $\mathrm{mis}_{\triangle,t}(n)$ of maximal independent sets in n-vertex triangle-free graphs containing no induced matching of size $t + 1$.

Set families with forbidden inclusion patterns

Dániel T. Nagy
Rényi Institute, Hungary

Let $\mathcal{F}$ be a family of sets and P be a partially ordered set (poset). We say that $\mathcal{F}$ is P-free if there is no injective map $f : P \to \mathcal{F}$ such that $f(a) \subset f(b)$ holds for all pairs $a, b \in P$, $a < b$. The problem of finding the largest P-free family formed by subsets of $[n] = \{1, 2, \ldots, n\}$ is solved for certain classes of posets, but open for general P.

We will review some classic results and open questions from this topic, then discuss a variant of the problem asking for the largest number of k-chains $A_1 \subset A_2 \subset \cdots \subset A_k$ in a P-free family of subsets of $[n]$.

Complexity of total dominator coloring in graphs

KUSUM
Department of Mathematics, Indian Institute of Technology Ropar, Punjab, India

co-authors: Michael A. Henning, Arti Pandey, and Kaustav Paul

Let $G = (V, E)$ be a graph with no isolated vertices. A vertex v totally dominates a vertex w ($w \neq v$), if v is adjacent to w. A set $D \subseteq V$ called a *total dominating set* of G, if every vertex $v \in V$ is totally dominated by some vertex in D. The minimum cardinality of a total dominating set is the *total domination number* of G and is denoted by $\gamma_t(G)$. A total dominator coloring of G is a proper coloring of vertices of G, so that each vertex totally dominates some color class. The total dominator chromatic number $\chi_{td}(G)$ of G is the least number of colors required for a total dominator coloring of G. The TOTAL DOMINATOR COLORING problem is to find a total dominator coloring of G using the minimum number of colors. It is known that the decision version of this problem is NP-complete for general graphs.

We show that it remains NP-complete even when restricted to bipartite, planar, and split graphs. We further study the TOTAL DOMINATOR COLORING problem for various graph classes, including trees, cographs and chain graphs. First, we characterize the trees having $\chi_{td}(T) = \gamma_t(T) + 1$, which completes the characterization of trees achieving all possible values of $\chi_{td}(T)$. Also, we show that for a cograph G, $\chi_{td}(G)$ can be computed in linear-time. Moreover, we show that $2 \leq \chi_{td}(G) \leq 4$ for a chain graph G and give a characterization of chain graphs for every possible value of $\chi_{td}(G)$ in linear-time.

A brief history of the first semionline scheduling paper

M. Grazia Speranza
University of Brescia, Italy

In this talk, the concept of off-line and on-line optimization are introduced, together with the classes of algorithms used for their solution and the measures adopted to evaluate their effectiveness. In particular, the concept of competitive ratio is presented to assess the quality of an on-line algorithm. Then, the concept of semi-online optimization is introduced. Finally, the results obtained in the paper "Semi on-line algorithms for the partition problem", published in Operations Research Letters in 1997 and co-authored by H. Kellerer, V. Kotov, M.G. Speranza and Zs. Tuza, are presented.

2.2 Tuesday:

Some Conjectures and Results in Tilings

Peter Horak
University of Washington, USA

Tilings and tessellations belong to the oldest structures not only in geometry but in all mathematics. They have attracted the attention of best mathematicians. Even one of Hilbert's problems is on the topic. Tiling problems do not always have a geometric background, sometimes there is even an unexpected relation of a tiling to other parts of mathematics. For example, the roots of the Minkowski conjecture on tiling the n-space by unit cubes can be traced to geometry of numbers and to positive definite quadratic forms; Hao Wang's work on tilings has been inspired by decision problems; there is a well-known relation of Penrose tilings to crystallography, etc.

As a short historic introduction we present the conjecture of Minkowski. Its last open case was solved only in the previous year.

Our interest in tilings stems from coding theory, especially from the area of error-correcting codes in Lee metric. Therefore, in this talk we will focus on tiling the n-space by unit cubes or by a cluster (the union) of unit cubes; a special attention will be paid to the famous and long-standing Golomb-Welch conjecture.

Shortest paths in grids

Béla Vizvári
Department of Industrial Engineering, Eastern Mediterranean University

Grids are more general objects than geometric lattices. They occur in the nature as structures of crystals. These grids consist of polyhedrons that are in contact only with their faces and fill the space symmetrically without gaps. You can move between polyhedrons through the faces. The length of a step may depend on the passed face. The shortest paths between polyhedrons model the propagation of energy or other influences in crystals. However, grids can also be generalized to a higher dimension. In the most important part of the presentation, we will analyze the case of the 4-dimensional Body Centered Cubic (BCC) grid. Even higher dimensions are briefly discussed.

How to prove impossibility for packings of rectangles

Mihály Hujter

We provide a new proof of an old unpublished result (which was joint with Zsolt Tuza): Proving that n given rectangles cannot be packed into a big given rectangle must check at most $(n!)^2$ cases.

Optimal cutting arrangements in 1D

Attila Sali
Alfréd Rényi Institute of Mathematics and Department of Computer Science, BUTE, Hungary

co-author: Bowen Li, Carleton College, USA

Mathematical model of an industrial application is investigated. Orders with given tolerances must be fit on a warehouse of steel rods so that the number of cuts needed to satisfy the orders is minimized. It is shown that the problem of feasibility and if the orders can be satisfied, then finding the minimum number of cuts needed are both NP-complete. Two practical solution methods are introduced: one is based on dynamic programming and maximum clique search in graphs, the other one uses $0-1$-linear programming. Simulations show that the latter one is much more effective.

Approximation algorithms for the star packing problem

An Zhang
Hangzhou Dianzi University, China

Let i be a positive integer. An i-star is a complete bipartite graph $K_{1,i}$. A sequential star set is the one containing all i-stars with $1 \le i \le k$ or $1 \le i < \infty$. Given a graph $G = (V; E)$ and a set Φ of stars, a Φ-packing of G is a set of vertex-disjoint subgraphs of G such that each subgraph is isomorphic to some element of Φ. The maximum Φ-packing problem is to find a Φ-packing of G that covers the maximum number of vertices. It has been shown to be strongly NP-hard unless Φ is a sequential star set. If Φ consists of all i-stars with $i \ge t$ (> 1), then it is called the t^+-star packing problem. And if it consists of all i-stars with $1 \le i \le T$ but $i = t$ (> 1), then it is called the T/t-star packing problem.

In this talk, I will focus on the discussion of approximation algorithms for these two star packing problems. First, I give an $O(|V|^4)$-time local-search algorithm for the t^+-star packing problem and show its approximation ratio is between $\frac{t+1}{2}$ and $\frac{t+2}{2}$. Then, by adding more local-search strategies to the algorithm, I provide a 5/3-approximation algorithm for the case of $t = 2$. Finally, I describe the idea of the 10/7-approximation algorithm for the T/t-star packing problem with $T = 3$ and $t = 2$.

Joint replenishment meets scheduling

Péter Györgyi
Institute for Computer Science and Control, Hungary

co-authors: Tamás Kis, Tímea Tamási (SZTAKI), and József Békési
(University of Szeged)

We consider a combination of two classic problems of operations research: the joint replenishment problem and the single machine scheduling with release dates. In this problem, each job requires some resources and it can be processed on the machine after the required resources are replenished. One has to decide both about the replenishments and about the schedule on the machine. The sum of the replenishment cost and the scheduling cost is to be minimized. The former depends on the number of the replenishments (i.e., it is independent from the supplied amount), while the latter is a standard scheduling criteria like the total weighted completion time or the maximum flow time.

We survey the complexity results for the offline problem and present some online and semi-online algorithms for different variants. In the semi-online variants we have some limited information about the jobs, e.g., there is an upper bound on the time period between the consecutive release dates of the jobs.

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On variants of a load-balancing problem

EVELIN SZÖGI
Institute for Computer Science and Control, Hungary

co-authors: Péter Györgyi and Tamás Kis

The optimization of the simultaneous use of shared resources is a challenging problem with numerous practical applications. Suppose there is a common resource required by a set of jobs for their execution. Each job has a duration, and a set of feasible time intervals when it can be executed, and requires one unit from the resource throughout its execution. The objective is to minimize a separable convex function of the load of the resource throughout the scheduling horizon. In some applications, the time horizon may be partitioned into disjoint time intervals and for each time interval a distinct convex cost function may be given. Then the cost of a solution is determined as a sum of the costs computed separately for each interval with the corresponding convex cost function.

When the duration of each job is one time unit, and there is only one convex cost function, then the offline problem was investigated by e.g., Hajek et al. (1990), Harvey et al. (2006) and Burcea et al. (2016). In the talk, we present a new, easier proof of the correctness of the algorithm of Burcea et al. (2016) for solving the problem with unit-time jobs and a single convex cost function and we also provide an efficient implementation of their method. The model with unit-time jobs can be extended by allowing preemptive jobs of arbitrary sizes with several feasible time intervals for each job, and several convex cost functions. We reduce the problem to a minimum-cost-flow computation in an appropriate network. We show that if there is only one convex cost function in the input of the preemptive problem, then it suffices to optimize the $(\cdot)^2$ function to obtain a feasible solution optimal for any convex cost function, and this leads to more efficient algorithms.

This research has been supported by the TKP2021-NKTA-01 NRDIO grant on "Research on cooperative production and logistics systems to support a competitive and sustainable economy".

Disjunctive cuts for modeling $R^3 \to R$ piecewise linear functions

Péter Dobrovoczki
Institute for Computer Science and Control, Hungary

co-author: Tamás Kis

Modeling $\mathbf{R}^d \to \mathbf{R}$ piecewise linear functions with mixed integer linear programming is a challenging problem with several applications. Suppose there is a rectangular grid, a function f defined on the convex hull of the grid and a triangulation (simplicial decomposition) of the cells of the grid. Consider $\hat{f}$, a piecewise linear approximation of the function f, which is defined equal to f on the grid points and as the linear interpolation of f between grid points. The goal is to describe $\hat{f}$ with a linear inequality system. There exists such formulation for the case $d = 2$ by Huchette and Vielma (2019), however their method, as a whole, cannot be straightforwardly generalized to higher dimensions.

We present a general framework to model a certain class of disjunctive constraints. We characterize the facets of the corresponding convex hull of polyhedra and also describe an efficient separation algorithm. We apply these results along with an efficient representation of SOS2 sets to describe $\mathbf{R}^3 \to \mathbf{R}$ piecewise linear functions. We tested our method on a series of benchmark instances and we summarize our computational results.

This research has been supported by the TKP2021-NKTA-01 NRDIO grant on "Research on cooperative production and logistics systems to support a competitive and sustainable economy".

Sharp bounds on the Total Coalition Number

János Barát
Alfréd Rényi Institute of Mathematics, and University of Pannonia, Department of Mathematics, Hungary

co-author: Zoltán L. Blázsik, Alfréd Rényi Institute of Mathematics, and SZTE Bolyai Institute, Hungary

Let $G(V, E)$ be a finite, simple graph without isolated vertices. A set of vertices $T \subseteq V$ is a *total dominating set* if every vertex of V is adjacent to at least one vertex of T. The following notions were recently introduced by Alikhani, Bakhshesh and Golmohammadi. Two disjoint sets $A, B \subset V$ form a *total coalition* in G, if none of them is a total dominating set, but their union $A \cup B$ is a total dominating set. A vertex partition $\Psi = \{C_1, C_2, \ldots, C_k\}$ is a *total coalition partition*, if none of the partition classes is a total dominating set, meanwhile for every $i \in \{1, 2, \ldots, k\}$ there exists a distinct $j \in \{1, 2, \ldots, k\}$ such that C_i and C_j form a total coalition. The maximum cardinality of a total coalition partition of G is the *total coalition number* of G and denoted by $TC(G)$. We give a general sharp upper bound on the total coalition number as a function of the maximum degree. The *total coalition graph*, denoted by $TCG(G, \Psi)$, is created by associating the partition classes of a total coalition partition Ψ with the vertex set, and the edges correspond to those pairs of classes, which form a total coalition. We show that every graph can be realised as a total coalition graph. Numerous natural questions remain unsolved.

Lower Bound for 3 Batched Bin Packing

József Békési
Institute of Informatics, University of Szeged, Hungary

In this talk we consider a special relaxation of the well-known online bin packing problem. Items are presented in a constant number of batches, and each batch should be packed before the next batch is presented. The case of three batches are studied and we present a lower bound for the asymptotic competitive ratio (ACR) of any on-line batched bin packing algorithm.

Why implementation matters for graph algorithms?

Máté Hegyháti
Institute of Informatics and Mathematics, University of Sopron

co-author: Zsolt Tuza

With the continuous increase for cheaply available computational power in the last few decades, computer-aided enumeration and search became a common and widely used asset in research for significant graph structures. While asymptotic time complexity of algorithms is often emphasized, and rightfully so, less attention is given to the implementation details. However, the practical execution time can be significantly affected by the selection of the programming language, compiler, memory allocation, etc. Naturally, these decisions themselves cannot asymptotically improve the efficiency of a theoretically optimized algorithm, however, a small constant multiplier can have significant practical benefits. Moreover, a careless implementation can easily degrade the asymptotic runtime of a well designed algorithm.

The aim of this presentation is to highlight and emphasize the aforementioned phenomenon via an example on the search for non-C-perfect uniform mixed hypergraphs.

Betweenness Structures of Small Linear Co-Size

Péter G. N. Szabó
University of Pécs, Hungary

One can study the combinatorial structure of a metric space by studying the corresponding betweenness relation. We say that point y is between points x and z in a metric space $M = (X, d)$ if $d(x, y) + d(y, z) = d(x, z)$. In such a case, $\{x, y, z\}$ is called a collinear triple, or alternatively, a degenerate triangle of M. In the hypergraph metrization problem, one aims to find and characterize metric betweennesses where the degenerate triangles coincide with the edges of a given 3-uniform hypergraph. In the last decades, several kinds of triangle hypergraphs were investigated: Richmond and Richmond characterized linear betweennesses that correspond to the complete 3-uniform hypergraph, while Steiner triple systems and some other block designs were shown to be not metrizable by Chen and Beaudou et al. Many of these result can be generalized to almost-metric (pseudometric) betweennesses which are (not necessarily metric) betweennesses satisfying a set of simple axioms. We extend the boundaries of the hypergraph metrization problem by solving a completely new case: we characterize the largest nonlinear betweennesses –called quasilinear betweennesses– in both the metric and the almost-metric case. We also generalize this result to betweennesses satisfying certain hereditary properties, as well as to ones containing $2n - c$ non-degenerate triangles. The talk is based on the author's paper of the same title published in Discrete Applied Mathematics in 2022.

Linear-Fractional model for reducing inconsistency of pairwise comparison matrices

Anett Rácz
University of Debrecen, Faculty of Informatics, Hungary

Pairwise comparison matrices (PCMs) are often used in multicriteria decision making (MCDM). The design of effective and precise mathematical tools for prioritizing among competing alternatives in MCDM problems is a critical component in many modern applications connected to human activity, including manufacturing, service industry, surveys etc. The most critical part of this technique is the inconsistency, which emerges for logical reasons but can cause significant problems during the decision making process. Hence it is necessary to keep it below an acceptable threshold. We have studied various linear programming models that are used for reducing the inconsistency of PCMs by trying to stay close to the original matrix, and also aiming to reduce the inconsistency as much as possible. We have identified the practical problems that arise when these objectives are not considered simultaneously. We have developed a mixed-integer linear-fractional programming (MILFP) model that takes both mentioned goals as objectives by combining them into a linear-fractional objective function. We also provide the linear analogue of our MILFP model using an appropriate adaptation and combination of the Charnes-Cooper transformation and Glover's linearization scheme.

2.3 Wednesday:

Non-crossing paths in geometric graphs

Jan Kratochvíl
Charles University, Prague, Czech republic

co-authors: Phillip Kindermann, Giuseppe Liotta, and Pavel Valtr

We prove that any set of at least 10 points in general position in the plane supports 3 edge-disjoint Hamiltonian paths, each of which is self-non-crossing (edges of different paths may cross).

Minors and Locally Spanning Subgraphs

Jochen Harant
Technical University of Ilmenau, Germany

Results on the existence of various types of spanning subgraphs of graphs are milestones in structural graph theory and have been diversified in several directions. Here, we consider "local" versions of such statements. In 1966, for instance, D. W. Barnette proved that a 3-connected planar graph contains a spanning tree of maximum degree at most 3. A local translation of this statement is that if G is a planar graph, X is a subset of specified vertices of G such that X cannot be separated in G by removing at most two vertices of G, then G has a tree of maximum degree at most 3 containing all vertices of X.

Our results constitute a general machinery for strengthening statements about spanning subgraphs of k-connected graphs (for $1 \leq k \leq 4$) to locally spanning versions, i. e. subgraphs containing a set $X \subseteq V(G)$ of a (not necessarily planar) graph G in which only X has high connectedness. Given a graph G and $X \subseteq V(G)$, we define X-minors of G and show that G has a highly connected X-minor if $X \subseteq V(G)$ cannot be separated in G by removing a few vertices of G.

As a first set of applications, we use this machinery to create locally spanning versions of some well-known results about degree bounded subgraphs of graphs.

Winding number and circular 4-coloring of (signed) graphs

Reza Naserasr

co-authors: Anna Gujgiczer, Rohini S, and S Taruni

Concerning the recent notion of circular chromatic number of signed graphs, for each given integer k we introduce two signed bipartite graphs, each on $2k^2 - k + 1$ vertices, having shortest negative cycle of length $2k$, and the circular chromatic number 4.

Each of the construction can be viewed as a bipartite analogue of the generalized Mycielski graphs on odd cycles, $M_\ell(C_{2k+1})$. In the course of proving our result, we also obtain a simple proof of the fact that $M_\ell(C_{2k+1})$ and some similar quadrangulations of the projective plane have circular chromatic number 4. These proofs have the advantage that they illuminate, in an elementary manner, the strong relation between algebraic topology and graph coloring problems.

Steiner systems and configurations of points

Elena Guardo
Università di Catania, Italy

co-authors: E. Ballico, G. Favacchio, and L. Milazzo

In this talk, based on the publication in Designs, Codes and Cryptography Vol. 89, pages 199–219 (2021), we introduce special configurations of reduced points in Pn constructed from a Steiner System, combining Combinatorial Algebraic Geometry and Commutative Algebra. In particular, we associate two ideals, in a suitable polynomial ring, defining a Steiner configuration of points and its Complement. We focus on the Complement of a Steiner configuration of points since it is a proper hyperplanes section of a monomial ideal that is the Stanley-Reisner ideal of a matroid. This connection allows us to study the homological invariants (Hilbert Function and Betti numbers) of the ideal IX_C of the Complement of a Steiner configuration. We also compute the parameters of linear codes associated to any Steiner configuration of points and its Complement.

This talk is dedicated to Lucia Gionfriddo. and Lorenzo Milazzo. I also thank M. Gionfriddo, B. Harbourne, S. Milici and Zs. Tuza for their encouragement in finishing the project.

Sufficient conditions for the existence and uniqueness of the maximum likelihood estimation in the case of the Thurstone method

László Gyarmati
Department of Mathematics, University of Pannonia, Hungary

co-authors: Csaba Mihálykó and Éva Orbán-Mihálykó

In this presentation paired comparison models with stochastic background are investigated. We focus on the models which allow three options for choice and the parameters are estimated by maximum likelihood method. The existence and uniqueness of the estimator is key issue of the evaluation. Allowing two options, a necessary and sufficient condition is given by Ford in the case of the Bradley-Terry model. We generalize this statement for the set of strictly log-concave distribution. Although in the case of three options necessary and sufficient condition is not known, there are two different sufficient conditions which are formulated in the literature. In this presentation we compare these conditions, moreover we present their generalization which guarantee the existence and uniqueness of the estimator. The sufficiency of the condition has been proved but we conjecture that it is a necessary condition, too. The conjecture is supported by a large number of computer simulations.

Optimal subsets of pairwise comparisons – the GRAPH of graphs approach

Zsombor Szádoczki
Institute for Computer Science and Control (SZTAKI, ELKH), Hungary; and Department of Operations Research and Actuarial Sciences, Corvinus University of Budapest, Hungary

co-author: Sándor Bozóki

Pairwise comparisons are the basis of preference modelling, ranking, and multi-criteria decision-making (MCDM). We are focusing on incomplete pairwise comparison matrices using their graph representation. In this study the optimal subsets of comparisons – i.e., the ones that provide the closest logarithmic least squares weight vectors to the vectors calculated from the complete case – are determined for the given numbers of items to

compare and comparisons. Simulations are used to find the optimal subsets, which result in a GRAPH of graphs for a given number of alternatives. Regularity and bipartiteness are the most important properties of the optimal graphs, which can often be reached from each other by adding/deleting exactly one comparison. The sequences of comparisons gained this way can be particularly useful in those problems, when the number of comparisons provided by the decision-maker is uncertain (e.g., online questionnaires).

Application of the flexible input modeling technique for the P-Graph framework

András Éles
Department of Computer Science, University of Pannonia, Hungary

co-authors: Heriberto Cabezas (University of Miskolc) and István Heckl (University of Pannonia)

Our aim is to demonstrate how the P-Graph framework can be extended with flexible operating units and how they can be used for designing an energy system. The P-Graph framework is used for modeling and optimization of Process Network Synthesis problems. It is based on a bipartite graph where one partition of the nodes represents materials, while the other partition represents operations on the materials. The arcs correspond to consumption and production. Originally, the P-Graph framework was developed for the chemical industry but later it was applied to other areas. The interpretation of the material was also extended, it can be energy, time, or any other abstract concept. In this work, we present how the operation unit can be extended as well. Originally, an operating unit in a P-Graph model consumes a fixed proportion of each input material and produces a fixed proportion of each output material. In certain scenarios, the ratio of the input materials may vary, often according to specific limits. Also, input compositions may be restricted by arbitrary linear constraints. We introduce a method for modeling operations with variable input and output material ratios, solely based on the introduction of additional material and operating unit nodes. The resulting P-Graph model can be directly optimized with the original tools of the P-Graph framework. This technique is demonstrated with a motivational example, including a fermenter model with different biomass inputs to address an energy production problem in a rural area.

Comparison of Optimization Algorithms for the Dynamic Capacitated Arc Routing Problem

Zsuzsanna Nagy
University of Pannonia, Hungary

co-authors: Ágnes Werner-Stark and Tibor Dulai

The Capacitated Arc Routing Problem (CARP) is an NP-hard combinatorial optimization problem, in which a fleet of capacitated vehicles have to serve all the demands on a set of road segments of a road network (i.e., a set of arcs of a weighted graph). The optimal solution is the least cost service plan (i.e., a set of route plans) on the graph for the vehicles that meets all the constraints. In the Dynamic CARP (DCARP), the changes in the components of the problem during the execution of the service plan are considered and a new plan is created when it is necessary.

In the literature, the available optimization algorithms for the DCARP were tested only on simple DCARP scenarios. In order to give a more exhaustive evaluation of these algorithms, we tested them on more complex and realistic DCARP scenarios using our previously developed data-driven DCARP framework. In this talk, the efficiency of the algorithms will be compared based on the results of these tests.

On Proper Quasi-Connection of Digraphs

Elżbieta Sidorowicz
University of Zielona Góra, Poland

An arc-colored path is properly colored if it does not contain two adjacent arcs with the same color. An arc-colored digraph D is properly quasi-connected if, between every ordered pair of vertices, which are connected by a path in D, there is a properly colored path. In that case, we say that the corresponding arc-coloring is a proper quasi-connected arc-coloring of D. The proper quasi-connection number of D, denoted by $\overrightarrow{pqc}(D)$, is the minimum number of colors needed to color the arcs of D so that D is properly quasi-connected.

In this talk, we will present some properties of proper quasi-connected coloring. We will provide the proper quasi-connection number for certain classes of digraphs, and also give upper bounds on the proper quasi-connection number.

Domination, isolation and the Art Gallery Theorem

Peter Borg
Department of Mathematics, Faculty of Science, University of Malta

In 2017, Caro and Hansberg [6] introduced the isolation problem, which generalizes the domination problem. Given a graph G and a set $\mathcal{F}$ of graphs, the *$\mathcal{F}$-isolation number of G* is the size of a smallest subset D of the vertex set of G such that $G - N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. When $\mathcal{F}$ consists of a 1-clique, the $\mathcal{F}$-isolation number is the domination number. In addition to establishing many results on $\mathcal{F}$-isolation numbers, Caro and Hansberg [6] posed several problems. Solutions [1, 3] will be presented together with other results. Chvatal's Art Gallery Theorem (ALT) [7] inspired a wealth of domination results (many of which are referenced in [5, 8] for the case where G is a maximal outerplanar graph (mop). Recently, Kaemawichanurat and the speaker [5] improved Chvatal's sharp upper bound $n/3$ on the domination number of an n-vertex mop G, and by treating $\{K_{1,k}\}$-isolation of G, they improved ALT for the case where at least one of every $k+1$ consecutive corners of an 'art gallery' (a polygon in general) needs to be guarded.

[1] P. Borg, Isolation of cycles, Graphs and Combinatorics 36 (2020), 631–637.

[2] P. Borg, Isolation of connected graphs, arXiv:2110.03773.

[3] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques, Discrete Mathematics 343 (2020), paper 111879.

[4] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques II, Discrete Mathematics 345 (2022), paper 112641.

[5] P. Borg and P. Kaemawichanurat, Extensions of the Art Gallery Theorem, Annals of Combinatorics (2022), in press, https://doi.org/10.1007/s00026-022-00620-4.

[6] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, FiloMath 31:12 (2017), 3925–3944.

[7] V. Chvátal, A combinatorial theorem in plane geometry, Journal of Combinatorial Theory Series B 18 (1975), 39–41.

[8] M. Lemańska, R. Zuazua and P. Zylinski, Total dominating sets in maximal outerplanar graphs, Graphs and Combinatorics 33 (2017), 991–998.

List coloring solution of the Futoshiki puzzle

BANU BAKLAN SEN
Biruni University, Turkey

co-author: Oznur Yasar Diner, Kadir Has University, Turkey

Futoshiki puzzle is one of the Latin Square Completion-Type Puzzles that has been a focus of research in the last few decades with a variety of applications in optimization. The decision version of the Futoshiki puzzle, the Futoshiki problem, is in NP-complete. When the board size is fixed, we propose an efficient algorithm to solve the Futoshiki problem with a newly defined coloring problem called "List PreColoring Extension". To test the efficiency of the developed algorithm, computational experiments are performed in different difficulty levels using varying number of inequality constraints. Furthermore, we introduce a new application of the Futoshiki puzzle whose solution is optimizing the personnel scheduling problem.

Optimal transport flow distributions on dynamic networks

ARMEN BAGDASARYAN
American University of the Middle East, Kuwait

co-authors: Antonios Kalampakas and Mansoor Saburov

In 1952 John Wardrop formulated two principles of optimality of flows in networks that describe the user (Wardrop) equilibrium and the system optimum. The Wardrop equilibrium is an optimal flow distribution across alternative parallel links in the network that minimizes the effective costs of the links defined as the sum of the latency at the given flow and the price of the link, while the system optimum is the optimal flow distribution for which the average effective cost for all used links is minimal.

We study the *Wardrop optimal flows* that satisfy both principles, the Wardrop equlibrium and the system optimum. A network having Wardrop optimal flow is called a *Wardrop optimal network*. We investigate dynamic properties of the Wardrop optimal networks and examine the Wardrop optimal flows on networks of parallel links. We present a characterization of Wardrop optimal flows and provide a geometric description of the set of all Wardrop optimal networks with common Wardrop optimal flow. We propose a new dynamical model of optimal flow distribution using the ideas of evolutionary game theory by presenting a discrete-time replicator equation on Wardrop optimal network. We describe the dynamics, equilibrium and stability conditions of the replicator equation. The results can shed a new light on optimal transport problems related to optimal flow allocation in transportation and communication networks.

2.4 Thursday:

New Lower Bounds for Certain Online Packing Problems with Flexibility

János Balogh
Institute of Informatics, University of Szeged, Hungary

co-authors: György Dósa, Leah Epstein, and Lukasz Jez

Packing problems are optimization problems that involve finding efficient ways to arrange items into containers (bins). Online packing problems are more challenging, because the items arrive one by one and the decisions are irrevocable. We study two online packing problems that have some flexibility in their constraints: online removable multiple knapsack and online minimum peak appointment scheduling.

In the online removable multiple knapsack problem, we have a fixed number of bins with capacity 1 each, and a sequence of items with sizes and values that arrive online. The goal is to maximize the total value of the items that are packed in the bins. An online algorithm can reject an item without packing it, or remove a previously packed item at any time. We prove new lower bounds on the performance of online algorithms for this problem, using techniques such as adaptive item sizing and dealing with special small cases. We also compare our results with previous work on related online packing problems. In particular, we improve the lower bounds for deterministic and randomized algorithms for the proportional and unit cases, where the value of an item is proportional (e.g. equal to its size) or identical.

We also study a particular online packing problem called online minimum peak appointment scheduling. In this problem, we have an infinite number of bins with capacity 1 each, and a sequence of items with sizes at most 1 that arrive online. An online algorithm must assign a position to each item upon its arrival, where the position is an interval of length equal to the item's size within a bin. The objective function used is determined by the maximum number of overlapping intervals at any point in time.

The goal is to minimize this objective, when we pack all the items. This problem models situations where there is some flexibility in choosing the positions of the items, such as appointment scheduling or task allocation. We prove new lower bounds on the performance of online algorithms for this problem.

This talk is based on a paper was presented at the 33rd International Workshop on Combinatorial Algorithms (IWOCA'22).

The research of J. Balogh has been supported by the grant TKP2021-NVA-09 of the Ministry for Innovation and Technology, Hungary.

Uniquely restricted generalized matchings

Miklós Krész
InnoRenew CoE & UP IAM, and University of Szeged

A matching M is called uniquely restricted if there is no other matching covering the same set of vertices. The concept was introduced by M. C. Golumbic, T. Hirst and M. Lewenstein in 2001. They have proved that finding a maximum uniquely restricted matching is NP-hard; consequently research in the recent years mainly focused on special cases of this problem. Another approach was considered by V. E. Levit and E. Mandrescu in a paper from 2003, when posed the question whether it is polynomially solvable for a graph G, if all maximum matchings are uniquely restricted. For the above question Penso et al gave recently (Journal of Graph Theory, 2018, Vol. 89) a positive answer. In this talk we are extending the concept for generalized matchings. We will consider (g, f)-factors as a classical generalization, but by the same way the concept can be further extended (e.g. matroid parity).

Given an undirected graph G, let g and f be nonnegative integer-valued functions defined on the vertex set V of G with $g(v) \leq f(v) \leq deg_G(v)$ for all $v \in V$, where $deg_G(v)$ represents the degree of vertex v in G. Then a (g, f)-factor is defined as a subgraph H of G with $g(v) \leq deg_H(v) \leq f(v)$ for every $v \in V$. One of the relevant characteristics of a (g, f)-factor is the so-called *degree pattern*: a vector p_H of the degrees of the vertices with a preliminary fixed order of the vertices. A (g, f)-factor H is *uniquely restricted* if the degree pattern of any other (g, f)-factor in G is different from p_H.

In this talk we will show that it is decidable in polynomial time if all (g, f)-factors of a graph G are uniquely restricted.

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Optimization of tree bucking with quality considerations

Balázs Dávid
InnoRenew CoE and University of Primorska, Slovenia

Tree bucking, the process of cutting tree stems into logs, plays an important role in the transformation of harvested trees into lumber products. While different variations of the problem class have been studied in the past, none have dealt with the case where segment-wise quality information is available for the tree stems. This presentation introduces a mathematical model for bucking a set of non-uniform tree stems into several log types with regards to the quality requirement and demand of each type, minimizing the leftover volume of the cut trees.
The author gratefully acknowledges the European Commission for funding the InnoRenew CoE project (Grant Agreement #739574) under the Horizon2020 Widespread-Teaming program, and the Republic of Slovenia (Investment funding of the Republic of Slovenia and the European Union of the European Regional Development Fund). He is also grateful for the support of the Slovenian National Research Agency (ARRS) through grants N1-0223 and V4-2124, and the University of Primorska postdoc grant No. 2991-5/2021.

2.5 Friday:

Various extensions on safe sets in graphs

Shinya Fujita
Yokohama City University, Japan

A non-empty subset S of the vertices of a connected graph G is a *safe set* if, for every connected component C of $G[S]$ and every connected component D of $G - S$, we have $|C| \geq |D|$ whenever there exists an edge of G between C and D. Since I introduced the concept of the safe set in a graph in [S. Fujita, G. MacGillivray, T. Sakuma: Safe set problem on graphs. Discret. Appl. Math. 215: 106-111 (2016)], this idea has gained attention and various related results have been demonstrated.

In particular, in a paper with Professor Tuza [R.B. Bapat, S. Fujita, S. Legay, Y. Manoussakis, Y. Matsui, T. Sakuma, Zs. Tuza: Safe sets, network majority on weighted trees. Networks 71(1): 81-92 (2018)], we obtained various results on the extension of this concept to weighted graphs as well as on algorithmic aspects in separate papers (see e.g. [R. Águeda, N. Cohen, S. Fujita, S. Legay, Y. Manoussakis, Y. Matsui, L. Montero, R. Naserasr, H. Ono, Y. Otachi, T. Sakuma, Zs. Tuza, R. Xu: Safe sets in graphs: Graph classes and structural parameters. J. Comb. Optim. 36(4): 1221-1242 (2018)]).

In this talk, some recent results in this topic will be reviewed.

Consecutive colourability of (oriented) graphs

Ewa Drgas-Burchardt
University of Zielona Góra, Poland

An oriented graph is consecutively colourable if it admits a proper arc colouring in which for each vertex the set of colours of in-arcs and the set of colours of out-arcs that are incident with the vertex are both intervals of integers. We analyse the existence of consecutively colourable orientations of graphs and its relationships with other graph properties.

Self-avoiding walks with wrong steps

László Szalay
Institute of Informatics and Mathematics, University of Sopron, Hungary
and Department of Mathematics, Jan Selye University, Slovakia

The investigation of self-avoiding walks on graphs has an extensive literature. Let n and m denote two positive integers. We study the notion of wrong steps of self-avoiding walks on rectangular shape $n*m$ grids of square cells (Manhattan graphs) and examine some general and special cases.

The number of self-avoiding walks with one and with two wrong steps are determined in general. Specific properties, like unimodality and sum of the rows of the Pascal-like triangles corresponding to the walks are also established.

The research was supported by National Research, Development and Innovation Office Grant 2019-2.1.11-TÉT-2020-00165, by Hungarian National Foundation for Scientific Research Grant No.~128088, and No.~130909, and by the Slovak Scientific Grant Agency VEGA 1/0776/21.

Special walks on $2 * n$ grids

László Németh
Institute of Informatics and Mathematics, University of Sopron, Hungary

The investigation of self-avoiding walks on graphs has an extensive literature. Let n and m denote two positive integers. We study the notion of wrong steps of self-avoiding walks on rectangular shape $n*m$ grids of square cells (Manhattan graphs) and examine some general and special cases.

We present particular recurrence relations on the number of self-avoiding walks on the $n * 2$ square grid in case of any w wrong steps. The Pascal-like triangle involving these self-avoiding walks is established. A special grid transformation provides the opportunity to give results for self-avoiding walks with wrong steps on hexagonal grids.

The research was supported by National Research, Development and Innovation Office Grant 2019-2.1.11-TÉT-2020-00165.

Boundedness in the structure of the set of minimal-length slice rank decompositions of a tensor over a finite field

Thomas Karam
University of Oxford, United Kingdom

If k is a nonnegative integer and $\mathbb{F}$ is a field, then all length-k decompositions of a matrix with rank k over $\mathbb{F}$ can be obtained from one another after a change of basis. In particular, if the field $\mathbb{F}$ is finite, then the number of such decompositions can be expressed in terms of k and $|\mathbb{F}|$, and is at most $|\mathbb{F}|^{k^2}$. More generally, if $d \geq 2$ is an integer, then the number of length-k tensor rank decompositions of an order-d tensor with tensor rank k is at most $|\mathbb{F}|^{(d-1)k^2}$. I will explain how an analogous statement can be obtained for the slice rank, another notion of rank on tensors introduced by Tao in his reformulation of the central idea from the solution to the cap-set problem by Croot, Lev, Pach, Ellenberg and Gijswijt, and which has since then had several further applications in combinatorics: although the number of length-k slice rank decompositions of an order-d tensor with slice rank k is no longer bounded above in terms of d,k and $|\mathbb{F}|$, this is nonetheless the case up to a class of transformations which can be described in a simple way.

A (1.999999)-approximation ratio for vertex cover problem on graphs with MaxCut value smaller than $0.9|E|$

M. Zohrehbandian
Karaj Branch, Islamic Azad University, Karaj, Iran

co-authors: N. Yekezare and M. Maghasedi

Vertex cover problem is a famous combinatorial problem, which its complexity has been heavily studied over the years and it is known that there is not any mathematical programming formulation that approximates it better than $2 - o(1)$, while a $2-$approximation for it can be trivially obtained. In this paper, by a combination of a well-known semidefinite programming (SDP) formulation along with satisfying new proposed properties, we introduce an approximation algorithm for the vertex cover problem with a performance ratio of 1.999999 on graphs with MaxCut value smaller than $0.9|E|$.

To do this, we show that there is a $(2 - \epsilon)$-approximation ratio for the vertex cover problem, based on any VCP feasible solution, where the value of ϵ is not constant and its value depends on the produced feasible solution. Then, we fix the ϵ value equal to 0.000001 and we show that, if the maximum cut value on G is less than $0.9|E|$ then the solution of the SDP relaxation satisfies some of the desired conditions and based on such a solution, we have an approximation ratio of less than 1.999999.

Finally, we propose an idea to extend the approach for graphs with maximum cut values more than $0.9|E|$, and this may lead to introduce a $(2 - \epsilon)$-approximation algorithm for vertex cover problem on all graphs.

Algorithm for solving a multiobjective optimization problem on a set of partial permutations

Serhii Khovben
Poltava University of Economics and Trade, Ukraine

co-author: Liudmyla Koliechkina, University of Lodz, Poland

The main idea of the algorithm is to use the properties of a combinatorial set of partial permutations, a polyhedron of partial permutations, and the graph of this polyhedron. The algorithm reduces the search area of the initial linear problem to solutions of a series of auxiliary combinatorial problems and performs branch pruning based on the properties of linear objective functions on special graphs associated with this problem.

The algorithm for solving discrete problems of combinatorial optimization is based on a combined approach and iteratively implements:

1) transition from a multiobjective problem to a linear optimization problem using linear convolution of criteria;

2) transition from a common set of combinatorial set of partial permutations to a partial combinatorial set, which is a subset of the original one; this subset is represented as a structured graph;

3) solving a linear problem on the constructed graph of a combinatorial set of partial permutations.

The proposed algorithm for solving linear optimization problems belongs to the group of well-known branch and bound methods. Such methods are successfully used to solve a number of applied problems.

On Nash-solvability of n-person graphical games under Markov and a-priori realizations

MARIYA NAUMOVA
Rutgers Business School, Rutgers University, Piscataway, New Jersey, United States

co-author: Vladimir Gurvich

We consider n-person games with perfect information on graphs. The games do not have Nash equilibria in pure stationary strategies. Solving these games in stationary mixed strategies as optimization problems, we introduce probability distributions in all non-terminal positions. The corresponding plays can be analyzed under two different basic assumptions: the Markov and a-priori realizations. The former one guarantees existence of a uniform best response for each player in every situation. Nevertheless, Nash equilibrium may fail to exist even in mixed strategies. The classical Nash's theorem is not applicable, since Markov realizations may result in discontinuous limit distributions and expected payoffs. Although a-priori realizations does not share many nice properties of Markov realizations (for example, the existence of uniform best responses) but in return, Nash's theorem is applicable. We illustrate both realizations in details by two examples with 2 and 3 players. We also survey some general results related to Nash-solvability, in pure and mixed stationary strategies, of stochastic n-person games with perfect information and n-person games on graphs among them.

3 List of Speakers